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HANANO Puzzle is NP-hard ☆

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ABSTRACT

We show that the HANANO Puzzle, a side-viewed 2-dimensional combinatorial puzzle with gravity and colored blocks, is NP-hard.

1. Introduction

Many combinatorial puzzles with simple rules are NP-hard [1–3]. We add HANANO Puzzle to the list of NP-hard puzzles in this paper. HANANO Puzzle literally means the puzzle of flowers in Japanese. It is a side-viewed 2-dimensional puzzle with gravity created by Qrostar in 2011 [4] for Windows system. A sequel Hanano Puzzle 2 with a new set of 35 levels was released in 2017 [5]. A screenshot of the puzzle is shown in Fig. 1.

The computational complexity of some other combinatorial puzzles with gravity has been studied before. A level of the computer puzzle Clickomania is a rectangular grid full of colored blocks. The player clicks a group of at least two connected blocks of the same color at each step. The selected group of blocks vanish and the blocks above fall down. The Clickomania problem asks whether the player can remove all the blocks by a finite number of clicks. It was shown that the decision problem for Clickomania is in P for one-column grids, and is NP-complete for either 5 colors and 2 columns or 3 colors and 5 columns [6] in 2002. Fifteen years later, it was proved that Clickomania is NP-complete for just 2 colors and 2 columns [7].

Puzznic (also known as Cubic) is another related puzzle. A level of Puzznic is a side-viewed 2-dimensional maze with several colored unit blocks in it, with no adjacent blocks with the same color initially. The player repeatedly choose to move a colored block one step to the left or to the right. After the move, all blocks without support fall due to gravity. And after the falling process comes to a stop, any group of connected blocks of the same color disappears. This may again cause a new round of falling and vanishing. The decision problem for Puzznic asks whether the player can remove all color blocks by a sequence of moves. Puzznic was proved to be NP-complete [8].

There are more puzzles of this type whose computational complexity has been studied, such as Bejeweled [9] and Tow Dots [10].

In this paper, we continue this line of research by studying the computational complexity of a new puzzle with gravity, the HANANO Puzzle. The rules of HANANO Puzzle are described in details in Section 2. In Section 3, we show that the HANANO Puzzle is NP-hard by reducing from

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the circuit satisfiability problem (CircuitSat), even with just one color. We conclude by stating an open problem for further study in Section 4.

2. The Hanano Puzzle

As illustrated on the left Fig. 2, a typical level of Hanano Puzzle consists of an unmovable platform (light gray area), colored 1×1 flowers fixed to the platform, and colored 1×1 stones with triangle marks. Each stone has exactly one triangle mark. Both the flowers and the stones have one of three different colors – red, yellow or blue. There may be some additional movable gray blocks of any size or shape. The level in Fig. 2 has exactly one 1×2 gray block shown in double rectangles and dark gray color (not to confused with the light gray platform). During the gameplay, yet another type of blocks, the blossomed stones, will emerge. A blossomed stone is a 1×2 or 2×1 block created by a flower of the same color springing out of one side of a colored stone (Fig. 2, right).

There are two types of operations allowed by the player, and both operations can be applied to the stones, the blossomed stones, or the gray blocks. The first type of operation is to move a stone (blossomed or not) or a gray block one step to the left or to the right, see Fig. 3. The second type of operation is to swap two adjacent blocks (stone, blossomed stone or gray block) of width one, see Fig. 4.

Fig. 1. A Screenshot of Level 14 of Hanano Puzzle 2.

After a move or a swap operation, all blocks without support will fall until they land on something else. And after the falling process, all stones toughing a flower with the same color (including the flower part of a blossomed stone, but not the stone part) will blossom. That is, a 1×1 flower of the same color grows out of the stone in the direction indicated by the triangle mark, resulting in a 1×2 block or a 2×1 block depending on the direction of the triangle mark. Each stone will blossom at most once, so it will not become a 1×3 or 3×1 block. In Fig. 4, the red stone blossomed after the swap. Again, the blossoming process may cause chain reactions. For example, a blossoming stone can push other blocks.

Note that a stone will blossom only if there is enough extra empty space, otherwise the stone will remain in unblossomed status even if it is touching a flower with the same color. We make use of this feature in constructing the AND gadget and the XNOR gadget in the next section.

To solve a level of the Hanano Puzzle, the player has to make all stones blossom by a sequence of operations. So we pose the following decision problem.

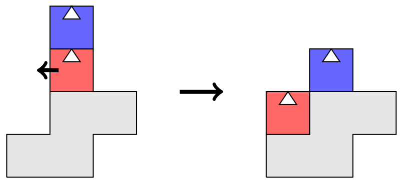

Fig. 3. A move to the left, followed by the falling of two stones.

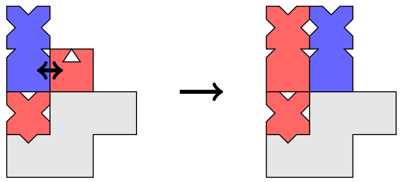

Fig. 4. A swap of two blocks, followed by the blossom of the red stone.

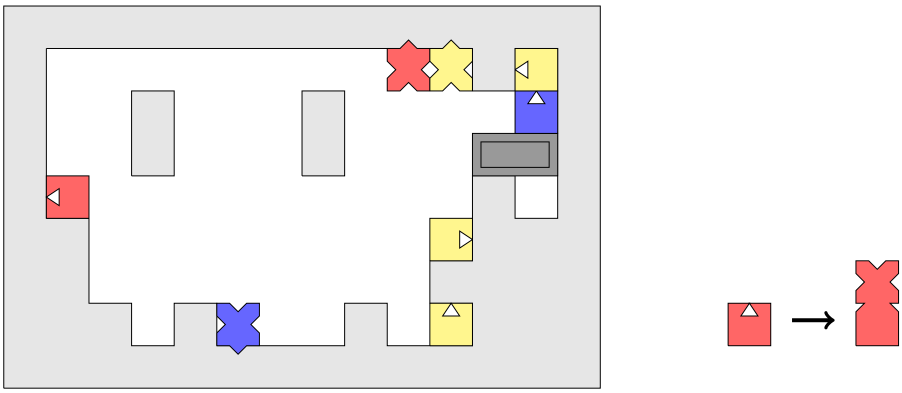

Fig. 2. A simple representation of Level 14 (left), and a blossomed stone (right). (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

Definition 1 *(***HANANO problem***).*
INSTANCE. A level of the HANANO Puzzle.
QUESTION. Is the level solvable?

Our main result is the following theorem.

Theorem 1. *The HANANO Puzzle is NP-hard, even with the following restrictions: (1) there are flowers and stones of just one color, (2) there are no gray blocks, and (3) all the stones would blossom upward.*

In the next section we will show that the HANANO Puzzle is NP-hard by a reduction from the CIRCUITSAT problem. The boolean circuit is an important model for the theory of computation and complexity, and the CIRCUITSAT problem is NP-complete [11,12].

Definition 2 *(***CIRCUITSAT problem***).*
INSTANCE. A boolean circuit with several inputs and one output.
QUESTION. Is there a TRUE/FALSE assignment for its inputs such that the output is TRUE?

3. The NP-hardness

Proof of Theorem 1. We first describe a few gadgets which emulate the input variables, the logic gates and the output value, or help connecting other gadgets. We show that these gadgets work as intended. Then we show that given an instance of the CIRCUITSAT problem, these gadgets can be put together to form a complete level of the HANANO Puzzle which emulates the circuit such that the level is solvable if and only if the circuit is satisfiable.

Each variable of the circuit is emulated by a variable gadget, as illustrated in Fig. 5. The red stone can blossom within the gadget by moving one step to the right, indicating the variable is assigned to the value FALSE. In other words, FALSE means no red stone is leaving the gadget. The red stone can also move one step to the left and leave the gadget, indicating the variable is assigned to the value TRUE.

So the TRUE or FALSE signal of the circuit is emulated by sending exactly one or no red stone along a wire. The SPLIT gadgets introduced later multiply stones if wires split. And all the gadgets are carefully designed so that at most one red stone is going out of each output wire. Therefore, it will never happen that two stones enter one input wire of some gadget.

The NOT gadget is shown on the left of Fig. 6. It has one entrance and one exit that can be connected to an input wire and an output wire, respectively. If an alien red stone is falling from the upper entrance, then the native red stone within the gadget has to move one step to the left so that the alien red stone can blossom (Fig. 6, middle). Then the blossomed stone moves one step left and falls, and the subsequent chain reactions cause the other stone to blossom (Fig. 6, right). As a result, no red stone will leave the gadget. On the other hand, if no alien red stone enters the gadget from above, then the native red stone must move to the right and leave the gadget. Note that by the rule of HANANO Puzzle, a stone does not blossom during the falling process even if it passes next to a flower with the same color, so the native red stone has no way to blossom inside the gadget by itself.

The AND gadget (see Fig. 7) has two entrances. As we mentioned earlier, by properly connecting each entrance to the exit of some other gadget, at most one red stone

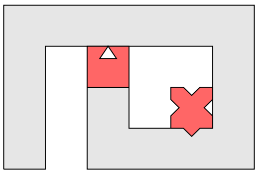

Fig. 5. The variable gadget.

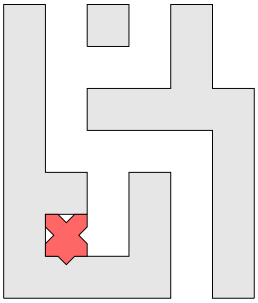

Fig. 7. The AND gadget.

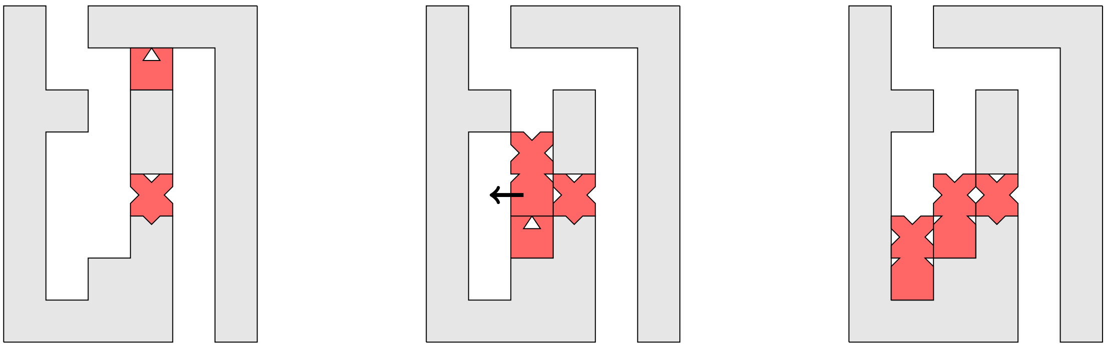

Fig. 6. The NOT gadget (left), and the case that a stone enters the NOT gate (middle, right).

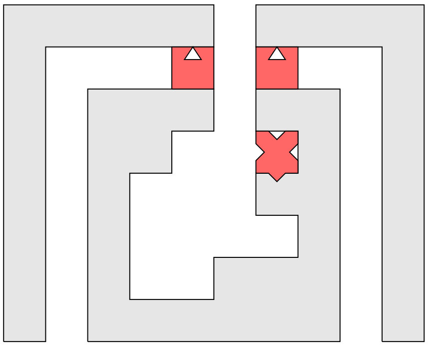

Fig. 8. The negation of a SPLIT gadget.

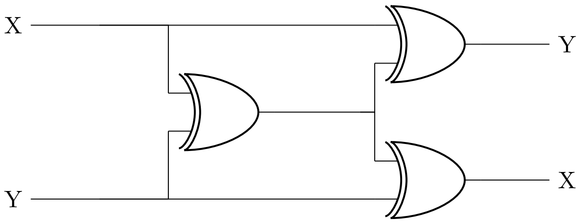

Fig. 9. CROSSING gadget by a planar layout of three XOR gates.

can fall into each of the two entrances. A red stone leaves the gadget if and only if there are stones entering both entrances. The second red stone cannot blossom inside the gadget because of the lack of space.

By the De Morgan's law, the OR gadget can be constructed by the combination of one AND gadget and three NOT gadgets.

We still need two more accessorial gadgets.

The SPLIT gadget splits a signal, TRUE or FALSE, into two. Fig. 8 shows the negation of a SPLIT gadget. If no red stone enters the gadget, the two red stones inside the gadget have no way to blossom, and have to leave the gadget from the lower left and lower right respectively. If a red stone enters the gadget, then the two native red stones must fall into the central chamber to allow the alien stone to blossom. So no red stones leaves the gadget in this case. This is actually similar to the NOT gadget except that we now need two stepping stones to blossom inside the central chamber. A combination of the NOT gadget and the gadget shown in Fig. 8 forms a SPLIT gadget as desired.

Another difficulty in emulating circuit with puzzles is how to emulate crossing wires. A technique [13] to overcome this obstacle is to use the planar layout of a circuit of three XOR gates, see Fig. 9. It is easy to check this circuit swaps the values of X and Y. So the problem is reduced to the emulation of the XOR gate by Hanano Puzzle. The XOR gate can be implemented by a planar circuit that uses only the AND gates and the NOT gates, both of which have already been described. What follows is a more direct emulation of the XOR gate. We will emulate XNOR gate instead, and the XOR gate is obtained by the combination of a XNOR gate and a NOT gate.

The XNOR gadget is illustrated in Fig. 10. If no alien red stone enters the gadgets, the native red stone has to leave.

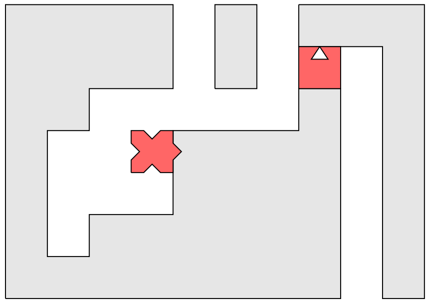

Fig. 10. XNOR gadget.

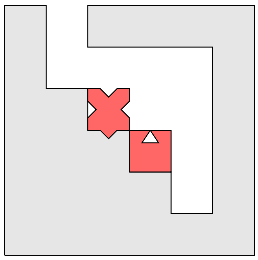

Fig. 11. The OUTPUT gadget.

If exactly one red stone enters, the native red stone must move left to help the alien stone, so no red stone leaves. If two red stones enter, then exactly one stone must leave the gadget since there is no space for a third stone to blossom inside.

The final output will be connected to the OUTPUT gadget (see Fig. 11). The native red stone can blossom if and only if a red stone enters the gadget, which in turn is equivalent to the circuit being satisfiable.

It is easy to see the above gadgets can be combined to emulate any instance of the CircuitSat problem. Due to gravity, the gadgets must be arranged in a way that signals are transmitted in the top-to-bottom direction. This can always be done because the circuit can be viewed as a directed acyclic graph. Note also that all the gadgets have fixed sizes, so the overall size of the Hanano level is a polynomial of the size of the circuit being emulated.

In all our gadgets, there are only one color of blocks, no gray blocks, and all stones blossom upwards. This completes the proof. □

4. Conclusion

We have shown that the 1-color Hanano Puzzle is NP-hard. It is not clear whether the associated decision problem for 1-color Hanano Puzzle is in NP. For many other combinatorial puzzles, if a puzzle is solvable, then it has a solution with at most polynomial number of steps with respect to the size of the puzzle. Therefore it is trivial to see these puzzles are in NP. But one is able to construct instances of Hanano Puzzle in which the number of moves in shortest solution is exponential with respect to the size of the instances, see Fig. 12 for an example with a lot of gray blocks. So it would be interesting to decide whether Hanano is in NP for future work.

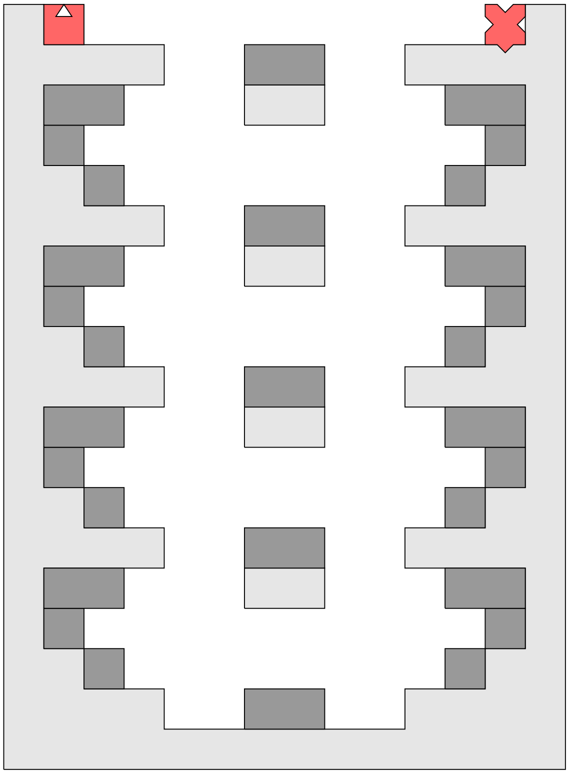

Fig. 12. Example with exponential solution.

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